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NUMERICAL CALCULATION OF FLOWS AND LONG-RANGE TRANSPORT OF CONTAMINANTS IN LOWLAND RIVER RESERVOIRS

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Flows and contaminant transport in the Novosibirsk reservoir are calculated on the basis of a two-dimensional (plane) nonstationary model with Saint Venant's equations. The model allows for the presence of a large number of islands. Coefficients of horizontal exchange (dispersion) are calculated by the formula taking into account dynamic velocity at the bottom. Numerical implementation of the model employs a semi-implicit conservative finite-difference TVD *scheme on a distributed grid and procedures allowing for the flow past these islands. Model examples of calculations and computation results for dynamics of long-range transport of contaminants along the Novosibirsk reservoir are given.*

Key words: *lowland reservoirs, contaminant, plane model,* TVD *schemes, predictor–corrector schemes.*

Introduction. Modern hydrodynamic calculations widely employ two-dimensional (plane) models based on Saint Venant's equations with model variables averaged over the depth of the water reservoir [1, 2]. Exact solutions of these equations are studied in [3]. Plane models were used in [4] for calculating flows and contaminant transport in the appurtenant zone of the Novosibirsk reservoir. The contaminant entered the computational domain with the waters of the main tributary of the river Berd'. It was numerically discovered that wind-driven circulations exceed the computational domain (appurtenant zone); therefore, the influence of the circulation calculation accuracy on numerical simulation of transport processes remained unclear. Thus, Zonov et al. [5] treated the flow over the entire reservoir with allowance for local transport of the contaminant brought in with the Berd' river waters.

The present study deals with long-range transport of contaminants along the reservoir. As distinct from [4, 5], the computational algorithm is based on an improved-accuracy difference scheme for modeling long-range transport of contaminants. It should be taken into account that the formal increase in the approximation order of advection terms of equations yields oscillating numerical solutions in the areas of drastic changes in gradients. It was shown in [6] that there are no monotonic finite-difference schemes with constant coefficients higher than the first order of approximation for a linear transport equation. This fact stimulated the development of solution-adaptive numerical algorithms securing monotonicity of the numerical solution and a high order of the numerical scheme (TVD, TVB, ENO schemes, etc.). A detailed procedure for construction of such algorithms and a description of their properties are given, e.g., in [7].

To improve the accuracy of the scheme we employ here, the well-known approach of Osher and Chakravarthy [8] was used, which guarantees the absence of increase in the total variation (TVD property) of the numerical solution with time for the case of scalar laws of conservation. This method is a generalization of the principle of minimum derivatives and can be used to construct schemes with the third-order approximations on smoothly varying solutions. Various modifications of these algorithms are widely used in modern methods of computational aero-hydrodynamics. For example, based on this algorithm, Kovenya and Karamyshev [9] proposed a conservative and easily implemented predictor–corrector scheme for gasdynamic equations, which yields sufficiently

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accurate numerical solutions of complex-structure discontinuities on rather coarse grids. Kovenya et al. [10] applied the construction procedure for Osher–Chakravarthy TVD schemes to schemes for solving spatial hydrodynamic problems for the real-configuration areas, such as hydroturbine ducts. In our study, this approach is used for numerical modeling of contaminant transport along lowland reservoirs.

1. Formulation of the Problem. The laws of conservation of momentum and mass and the equations of contaminant transport for depth-averaged turbulent flow can be written in the following form [2, 5]:

$$\frac{\partial Q_1}{\partial t} + \frac{\partial u_\alpha Q_1}{\partial x_\alpha} - lQ_2 = -gH\,\frac{\partial z}{\partial x_1} + \frac{\partial}{\partial x_\alpha}\,HK\,\frac{\partial u_1}{\partial x_\alpha} - \frac{r|Q|Q_1}{H^2},$$
$$\frac{\partial Q_2}{\partial t} + \frac{\partial u_\alpha Q_2}{\partial x_\alpha} + lQ_1 = -gH\,\frac{\partial z}{\partial x_2} + \frac{\partial}{\partial x_\alpha}\,HK\,\frac{\partial u_2}{\partial x_\alpha} - \frac{r|Q|Q_2}{H^2}; \qquad (1)$$

$$\frac{\partial z}{\partial t} + \frac{\partial Q_1}{\partial x_1} + \frac{\partial Q_2}{\partial x_2} = 0, \qquad \frac{\partial HC}{\partial t} + \frac{\partial Q_\alpha C}{\partial x_\alpha} = \frac{\partial}{\partial x_\alpha}\,HD\,\frac{\partial C}{\partial x_\alpha} - k_{\rm C}HC, \qquad \alpha = 1, 2. \qquad (2)$$

Here t is the time, z is the mark of water-surface deflection from its unperturbed state, $H(t, x_1, x_2)$ is the depth of the water reservoir, u_1 and u_2 are the components of the fluid velocity along the axes x_1 and x_2 respectively, l is the Coriolis parameter, r is the coefficient of friction on the bottom, K is the turbulent viscosity coefficient, D is the turbulent diffusion coefficient of the contaminant, $Q_i = Hu_i$ is the specific discharge, C is the contaminant concentration, and $k_{\rm C}$ is the contaminant decomposition coefficient.

The presence of horizontal turbulent exchange (longitudinal dispersion) in momentum equations (1) made it possible to impose boundary conditions containing turbulent or full momentum fluxes. The boundary conditions for system (1), (2) are written in the following form. No-slip conditions and the absence of turbulent momentum flux and turbulent contaminant flux are set on the solid boundaries (shores):

$$Q_{\rm n} = 0, \qquad HK\,\frac{\partial u_i}{\partial n} = 0 \quad (i = 1, 2), \qquad HD\,\frac{\partial C}{\partial n} = 0. \qquad (3)$$

Water discharge and the absence of turbulent momentum flux and turbulent contaminant flux are prescribed at the liquid boundaries where water flows out of the reservoir:

$$Q_{\rm n} = f_{\rm n}(x, t), \qquad HK\,\frac{\partial u_i}{\partial n} = 0 \quad (i = 1, 2), \qquad HD\,\frac{\partial C}{\partial n} = 0. \qquad (4)$$

Water discharge and the total turbulent momentum flux and total turbulent contaminant flux are set at the liquid boundary where water flows into the reservoir:

$$Q_{\rm n} = f_{\rm n}(x, t), \qquad Q_{\rm n}u_i - HK\,\frac{\partial u_i}{\partial n} = f_{{\rm n},i}(x, t) \quad (i = 1, 2), \qquad Q_{\rm n}C - HD\,\frac{\partial C}{\partial n} = f_{\rm C}(x, t). \qquad (5)$$

In (3)–(5), $\boldsymbol{n}$ is the vector of the normal to the shore contour, $Q_{\rm n}$ is the normal component of the vector of specific discharge, and $f_{\rm n}$, $f_{{\rm n},i}$, and $f_{\rm C}$ are given functions.

In addition to the boundary conditions (3)–(5), it is necessary to impose initial distributions of the sought parameters: $Q_1 = Q_1(x_1, x_2)$, $Q_2 = Q_2(x_1, x_2)$, $C = C(x_1, x_2)$, and $z = z(x_1, x_2)$ for $t = 0$.

2. Difference Equations. For the numerical solution of the formulated problem, the staggered Arakawa C-grid [11] is used. At the same time, the difference analogue of Eqs. (1) and (2) is written in the following form:

$$\frac{Q_1^{m+1} - Q_1^m}{\tau} = -gH\,\frac{\partial}{\partial x_1}\,\frac{z^{m+1} + z^m}{2} - \frac{r|Q|Q_1^{m+1}}{H^2} + \Lambda_1 u_1^m,$$

$$\frac{Q_2^{m+1} - Q_2^m}{\tau} = -gH\,\frac{\partial}{\partial x_2}\,\frac{z^{m+1} + z^m}{2} - \frac{r|Q|Q_2^{m+1}}{H^2} + \Lambda_2 u_2^m,$$

$$\frac{z^{m+1} - z^m}{\tau} + \frac{\partial}{\partial x_1}\,\frac{Q_1^{m+1} + Q_1^m}{2} + \frac{\partial}{\partial x_2}\,\frac{Q_2^{m+1} + Q_2^m}{2} = 0,$$

$$\frac{(HC)^{m+1} - (HC)^m}{\tau} = \Lambda_{\rm C}C^m - k_{\rm C}HC^{m+1}.$$

Here τ is the time step, m is the number of the time layer, Λ_1, Λ_2, and $\Lambda_{\rm C}$ are finite-difference operators for advective terms as well as for terms characterizing horizontal exchange and the Coriolis forces. The approximation of advective terms followed the above-mentioned Osher–Chakravarthy procedure of TVD scheme construction.

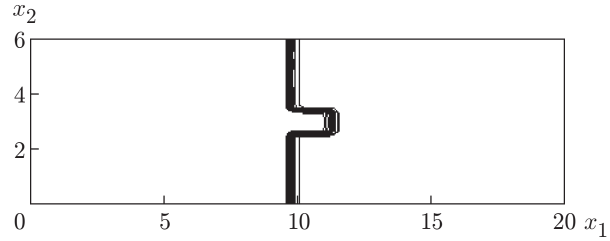

Fig. 1

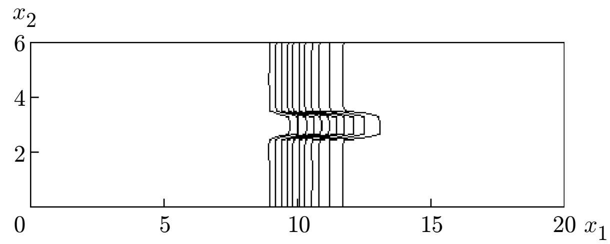

Fig. 2

The computational algorithm includes conversion (corrector) from the mth layer to the $(m+1)$th layer, providing the second order of time accuracy; at the same time, the values of parameters at the half-layer are determined as a half-sum of their values at the mth layer and the values at the $(m+1)$th layer obtained at the previous stage (predictor). The equation for the water-surface level is solved by the iterative over-relaxation method. The model allows for the complicated planform configuration of the water reservoir and for the presence of numerous islands.

3. Numerical Calculation Results. For testing the suggested algorithm, the problem of propagation of a conservative contaminant ($k_{\rm C} = 0$) in a channel (Fig. 1) was numerically solved. The channel was assumed to be 20 km long, 6 km wide, and 10 m deep. A uniform square grid with 200 m steps was constructed in the computational domain. The time step was assumed to be 200 sec.

First, the calculation was performed for a steady uniform flow in the channel with a velocity of 0.5 m/sec and specified water discharge $Q_1 = 5$ m^2/sec in the input (left boundary) and output (right boundary) sections. The coefficients of turbulent diffusion K and D were assumed to be identical ($K = D$) and equal to $6.2Hu_*$ [12] (u_* is the dynamic velocity at the bottom of the water reservoir). The background concentration of the contaminant was 5 μg/liter. Then, a local outburst of the contaminant was modeled in the middle part of the input section (band 800 m wide) where the contaminant concentration was specified to be 30 μg/liter. After 2800 sec, this condition was instantaneously transferred over the entire domain of the input section.

Figure 1 shows the calculation results for contaminant propagation at the time $t = 22\,800$ sec (114 time steps) obtained by the suggested predictor–corrector TVD scheme with second-order approximation. One can see the internal layers whose width reaches three difference intervals. Their position corresponds to the exact discontinuous solution of the problem in the absence of diffusion transport, which proves that the scheme is conservative. The nonoscillating behavior of the numerical solution in the area with singularities is also worth noting. Further grid refinement does not improve the computation accuracy. This shows that dissipative properties of the scheme do not prevail over the diffusional characteristics of the model.

Figure 2 shows the calculation results for the same problem obtained with the use of a first-order scheme. In contrast to Fig. 1, complete smearing of the solution structure (over 12 difference intervals) can be seen, which makes this algorithm inapplicable for calculating flows with drastic changes in gradients on relatively coarse grids. Such a scheme can hardly be applied to calculate viscous flows without additional algorithms for grid adaptation either.

The above-described model and numerical scheme were employed to calculate the flow and the processes of long-range transport of contaminants in the Novosibirsk reservoir from the input section of river Ob' near the city of Kamen'-na-Obi.

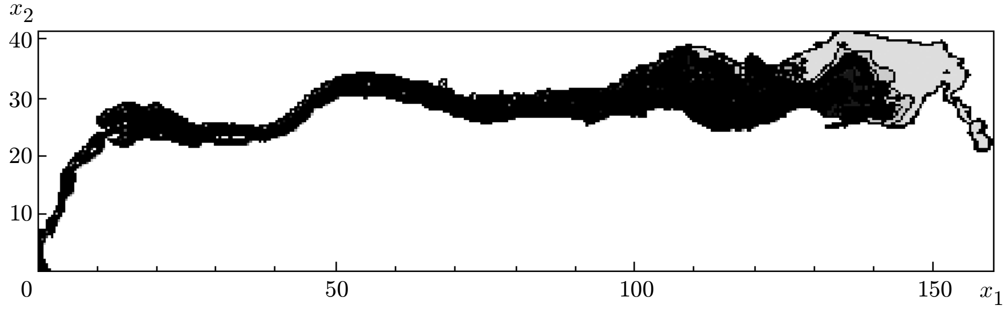

Fig. 3

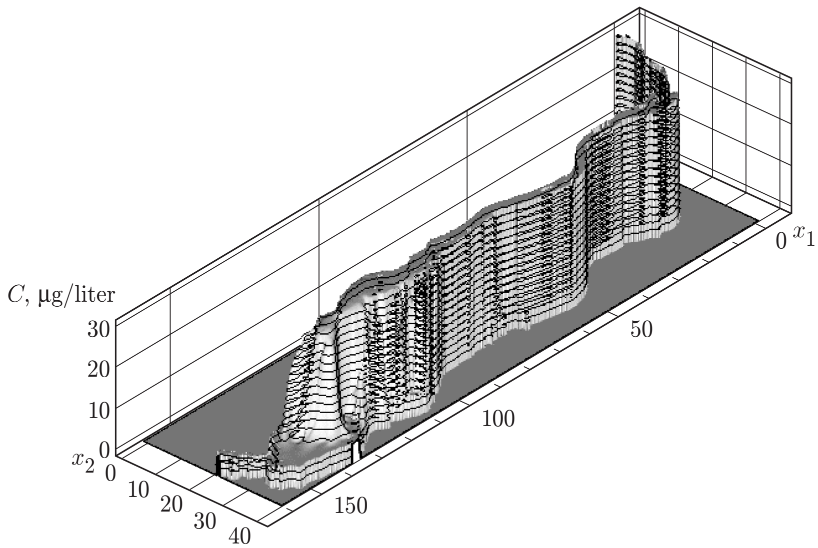

Fig. 4

The Novosibirsk reservoir, the largest in Siberia, is a fairly narrow oblong water reservoir (Fig. 3) [13] extending to nearly 180 km. The main parameters of the Novosibirsk reservoir are as follows [14]: the volume is 8.8 km^3, the mirror area is 1070 km^2, the maximum width is 17 km, the maximum depth is 22 m, and the average depth is 8.2 m. Over the distance from the city of Kamen'-na-Obi to the Zav'yalovo settlement (120 km), the reservoir looks like a river with a width up to 5 km. Over the range from the Zav'yalovo settlement to the dam, the reservoir is a lake-shaped reach. There are numerous islands along the left-shore flood-lands. The volume of water in the lake-shaped part of the reservoir is 73% of the total water volume. The main inflow of water into the Novosibirsk reservoir (over 95%) occurs through the head range along the Ob'. The lateral inflow accounts for less than 5% of the total inflow. The main part of the water inflow transits the reservoir. On average, flushing in the reservoir occurs seven times a year. According to the data of 1978, the maximum water discharge in the river Ob' (near the dam) is 3060 m^3/sec, and the corresponding value for the river Berd' is 169 m^3/sec (average annual water discharge in the river Ob' in 1978 is close to the mean annual water discharge) [15].

The time step in numerical calculations was 5 min, and the grid step was 300 m. The water discharge is 2250 m^3/sec in the input section of the Ob' (upstream of Kamen'-na-Obi), 2500 m^3/sec near the dam, and 250 m^3/sec in the river Berd'.

The discharge current was calculated starting from the state at rest with an initial contaminant concentration equal to 5 μg/liter. In the input sections (of the river Ob' near the city of Kamen'-na-Obi and in the mouths of the lateral tributaries of the Ob', including the river Berd'), background values of contaminant concentration equal to 5 μg/liter were also specified. The analysis of calculated time changes in the mean kinetic energy showed that the steady-state discharge current was attained within 3–4 days.

The dynamics of contaminant propagation in the reservoir was also studied as it was brought in with the Ob' river waters through the input section near the city of Kamen'-na-Obi with a constant concentration of 30 μg/liter. Figures 3 and 4 show the isolines and the contaminant-concentration distribution in 24 days after it entered the reservoir. It can be seen in Figs. 3 and 4 that the concentration remains unchanged over the entire extent of water space from the city of Kamen'-na-Obi to the middle of the lake part of the reservoir, and the fore front is of insignificant width. Similar calculations with the first-order scheme yielded front smearing of over 20 km. The contaminant distribution along the reservoir has a structure of a discontinuous "wave." The contaminant moves along the reservoir, retaining a constant concentration behind the "wave" front. The contaminant distribution is uniform over the width of the river part of the reservoir, and the nonuniform distribution with a higher contaminant concentration near the right shore of the lake part of the reservoir can be attributed to the character of the flow localized along the Ob' crease. The analysis of the contaminant-transport dynamics showed that, in the case of a disastrous contaminant inflow into the reservoir via the input section near Kamen'-na-Obi, it will take the contaminant about a month to reach the appurtenant zone.

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